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**BSc3MatC0301** **Seat No :_________**

# B.Sc. Semester - 3 (*CBCS*) Examination
## Oct/Nov. -2019 (New Course)
## MATHEMATICS (CORE)

**Time: 2:30 Hours** **Marks: 70**

**Instructions:**

1. All questions are compulsory.
2. Figures to the right indicate marks.

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**Q-1**

**(A)** **ATTEMPT ANY ONE:** **10**

**(1)** Define: Vector Space.
Prove that zero vector $\theta$ is unique in a vector space.

**(2)** Define: Subspace of a vector space.
If $W_1 \wedge W_2$ be any subspaces of a vector space $V$, then prove that $W_1 + W_2$ and $W_1 \cap W_2$ are also subspaces of $V$.

**(3)** Define: Linearly Independent Set & Linearly Dependent Set
Prove that a finite superset of a linearly dependent set of vectors is also linearly dependent.

**(B)** **ATTEMPT ANY ONE :** **04**

**(1)** Show that the set $V = \{(x, y) \in R^2 / y > 0\}$ is a vector space over $F = R$ under the vector addition and scalar multiplication defined as follows.

$$(a, b) + (c, d) = (a + c, bd), \forall (a, b), (c, d) \in V$$
and $$\alpha(a, b) = (\alpha a, b^\alpha), \forall (a, b) \in V, \forall \alpha \in R .$$

**(2)** Check linear dependence of the following sets:
(A) $\{e^{2x}, e^{4x}, e^{6x}\}$ (B) $\{sin(-x), cos(-x), sin\left(x + \frac{\pi}{6}\right)\}$

**Q-2**

**(A)** **ATTEMPT ANY ONE :** **10**

**(1)** State and prove the existence theorem for a basis of a finite dimensional vector space.

**(2)** Let $V$ be a finite dimensional vector space .
Then, prove that for every subspace $W_1$ of $V$, there exists another subspace $W_2$ of $V$ such that $V = W_1 \oplus W_2$.

**(3)** If $W_1 \wedge W_2$ be any two subspaces of a finite dimensional vector space $V$, then prove that $Dim(W_1 + W_2) = Dim(W_1) + Dim(W_2) - Dim(W_1 \cap W_2)$.

**(B)** **ATTEMPT ANY ONE :** **04**

**(1)** Show that the set $\{1 - x, 1 + x, 1 - x^2\}$ is a basis of $P_2(R)$. Also find the coordinates of the vector $1 + 2x + 3x^2$ w.r.t. this basis.

**(2)** Let $W_1 = \{(a, b, c, d) / a + c - d = 0\}$ & $W_2 = \{(a, b, c, d) / a + 2b = 0\}$ are subspaces of the vector space $R^4$.
Find $Dim(W_1), Dim(W_2) \wedge Dim(W_1 \cap W_2)$. Also, deduce that $W_1 + W_2 = R^4$.

**Q-3**

**(A)** **ATTEMPT ANY ONE :** **10**

**(1)** Define: Gradient
If $\phi$ and $\psi$ are differentiable scalar functions, then prove the followings:
(i)$\nabla(\phi\psi) = \phi\nabla(\psi) + \psi\nabla(\phi)$ (ii) $\nabla\left(\frac{\phi}{\psi}\right) = \frac{\psi\nabla(\phi) - \phi\nabla(\psi)}{\psi^2}$

**(2)** Define: (i) Divergence (ii) Curl
If $\bar{f}$ be a function whose second order derivative exists, then show that $\left(Curl(\bar{f})\right) = 0$.

**(3)** Define: Laplacian Operator

Prove that, $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $r = |\acute{r}|, \acute{r} = (x, y, z)$.

**(B) ATTEMPT ANY ONE:** **04**

**(1)** If $(x, y, z) = x^2y + xy^2 + z^2$ , then find $\nabla\phi$ and find unit normal at (1,2,3)**.**

**(2)** Show that the function $H = e^{-\lambda x}\left(c_1 sin(\lambda y) + c_2 cos(\lambda y)\right)$ satisfies the Laplace equation where $\lambda, c_1, c_2$ are constants.

**Q-4**

**(A) ATTEMPT ANY ONE:** **10**

**(1)** Define: Double Integration

Explain evaluation of double integrals and also explain change of order of integration.

**(2)** Define: Triple Integration

Explain change of variables in cylindrical variables and spherical variables in triple integration.

**(3)** Evaluate: $\iiint (a^2b^2c^2 - b^2c^2x^2 - c^2a^2y^2 - a^2b^2z^2) dxdydz$ over R, where R is ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

**(B) ATTEMPT ANY ONE:** **04**

**(1)** Evaluate: $\iint xydydx$ over R ,

whrere R is the first quadrant of the circle $x^2 + y^2 = 9$.

**(2)** Evaluate: $\int_0^a \int_0^b \int_0^c (x + y + z)\, dxdydz$.

**Q-5**

**(A) ATTEMPT ANY ONE :** **10**

**(1)** State and prove the Green’s theorem.

**(2)** State Gauss divergence theorem and verify the theorem for $F = (4x, -2y^2, z^2)$ where R is $x^2 + y^2 = 4, z = 0\ \&\ z = 3$.

**(3)** Prove that $\beta(p, q) = \frac{\Gamma p \Gamma q}{\Gamma(p+q)}, where\ p > 0\ \&\ q > 0$.

**(B) ATTEMPT ANY ONE:** **04**

**(1)** Verify the Stoke’s theorem for $F = (2x - y, -yz^2, -y^2z)$ & S is the upper half of the unit sphere.

**(2)** Find the value of $\Gamma \frac{1}{4} \Gamma \frac{3}{4}$ .

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